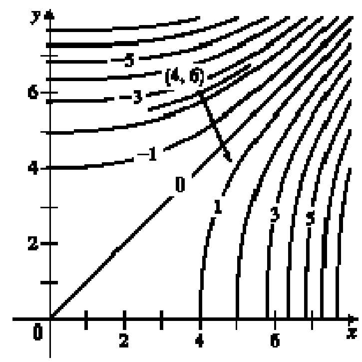
11.6: 34, 38, 42, 44, 46

34. If we place the initial point of the gradient vector $\nabla f(4,6)$ at $(4,6)$, the vector is perpendicular to the level curve of f that includes $(4,6)$, so we sketch a portion of the level curve through $(4,6)$ (using the nearby level curves as a guideline) and draw a line perpendicular to the curve at $(4,6)$. The gradient vector is parallel to this line, pointing in the direction of increasing function values, and with length equal to the maximum value of the directional derivative of f at $(4,6)$. We can estimate this length by finding the average rate of change in the direction of the gradient. The line intersects the contour lines corresponding to -2 and -3 with an estimated distance of 0.5 units. Thus the rate of change is approximately $\frac{-2-(-3)}{0.5} = 2$, and we sketch the gradient vector with length 2.

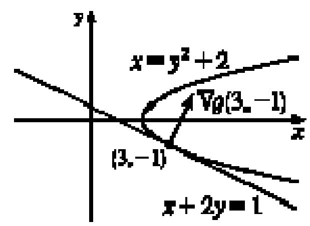


38. $F(x,y,z) = yz - \ln(x+z) \quad \Rightarrow \quad \nabla F(x,y,z) = \left\langle -\frac{1}{x+z}, z, y - \frac{1}{x+z} \right\rangle$, $\nabla F(0,0,1) = \langle -1, 1, -1 \rangle$.

(a) $(-1)(x-0) + (1)(y-0) - 1(z-1) = 0$ or $x - y + z = 1$.

(b) Parametric equations are $x = -t$, $y = t$, $z = 1 - t$ and symmetric equations are $\frac{x}{-1} = \frac{y}{1} = \frac{z-1}{-1}$ or $-x = y = 1 - z$.

42. $\nabla g(x,y) = \langle 1, -2y \rangle$, $\nabla g(3,-1) = \langle 1, 2 \rangle$. The tangent line has equation $\nabla g(3,-1) \cdot \langle x-3, y+1 \rangle = 0 \quad \Rightarrow$ $1(x-3) + 2(y+1) = 0$, which simplifies to $x + 2y = 1$.



44. Since $\nabla f(x_0,y_0,z_0) = \langle 2x_0, 4y_0, 6z_0 \rangle$ and $\langle 3, -1, 3 \rangle$ are both normal vectors to the surface at (x_0,y_0,z_0), we need $\langle 2x_0, 4y_0, 6z_0 \rangle = c\,\langle 3, -1, 3 \rangle$ or $\langle x_0, 2y_0, 3z_0 \rangle = k\langle 3, -1, 3 \rangle$. Thus $x_0 = 3k$, $y_0 = -\frac{1}{2}k$ and $z_0 = k$. But $x_0^2 + 2y_0^2 + 3z_0^2 = 1$ or $\left(9 + \frac{1}{2} + 3\right)k^2 = 1$, so $k = \pm\frac{\sqrt{2}}{5}$ and there are two such points: $\left(\pm\frac{3\sqrt{2}}{5}, \mp\frac{1}{5\sqrt{2}}, \pm\frac{\sqrt{2}}{5}\right)$.

46. First note that the point $(1,1,2)$ is on both surfaces. For the ellipsoid, an equation of the tangent plane at $(1,1,2)$ is $6x + 4y + 4z = 18$ or $3x + 2y + 2z = 9$, and for the sphere, an equation of the tangent plane at $(1,1,2)$ is $(2-8)x + (2-6)y + (4-8)z = -18$ or $-6x - 4y - 4z = -18$ or $3x + 2y + 2z = 9$. Since these tangent planes are the same, the surfaces are tangent to each other at the point $(1,1,2)$.